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2008 J. Phys.: Condens. Matter 20 204109

(http://iopscience.iop.org/0953-8984/20/20/204109)

IOP PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **20** (2008) 204109 (4pp) doi:10.1088/0953-8984/20/20/204109

Response of a ferrofluid to traveling-stripe forcing

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Received 1 April 2008
Published 1 May 2008
Online at stacks.iop.org/JPhysCM/20/204109

Abstract
We observe the dynamics of waves propagating on the surface of a ferrofluid under the influence of a spatially and temporally modulated field. In particular, we excite plane waves by applying a traveling lamellar modulation of the magnetization. By means of this external driving, both the wavelength and the propagation velocity of the waves can be controlled. The amplitude of the excited waves exhibits a resonance phenomenon similar to that of a forced harmonic oscillator. Its analysis reveals the dispersion relation of the free surface waves, from which the critical magnetic field for the onset of the Rosensweig instability can be extrapolated.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

When a critical value of the vertical magnetic induction is surpassed, the surface of a ferrofluid exhibits an array of liquid crests. This so called Rosensweig instability (Cowley and Rosensweig 1967) has been investigated in a static field (see e.g. Bacri and Salin 1984, Richter and Barashenkov 2005, Gollwitzer *et al* 2007) and under temporal modulation of either the magnetic field (see e.g. Mahr and Rehberg 1998) or the gravitational acceleration (Ko *et al* 2003). For a recent survey see Richter and Lange (2008). While these excitations are homogeneous in space, a combined spatiotemporal forcing of the plane surface has been implemented by Kikura *et al* (1990) using an array of solenoids. They investigate the surface waves for small magnetic fields far below the Rosensweig threshold and measure the resulting volume flow rate. In contrast, we apply a traveling-stripe forcing to the surface of magnetic liquids at the advent of the Rosensweig instability and uncover a resonance phenomenon of the wave amplitude with respect to the lateral driving velocity. So far, such a spatiotemporal driving of a pattern forming system has only been realized in a chemical experiment by Miguez *et al* (2003). A review by Rüdiger *et al* (2007) calls for further experiments, and the present paper provides one.

2. Experimental details

Our experimental setup is sketched in figure 1. A rectangular vessel machined from Perspex™ is placed in the center. The

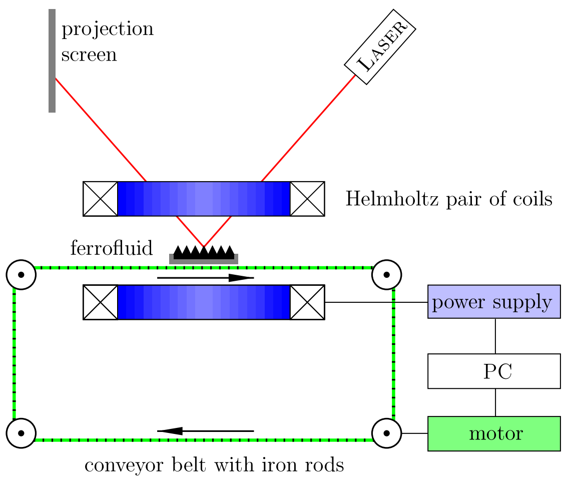
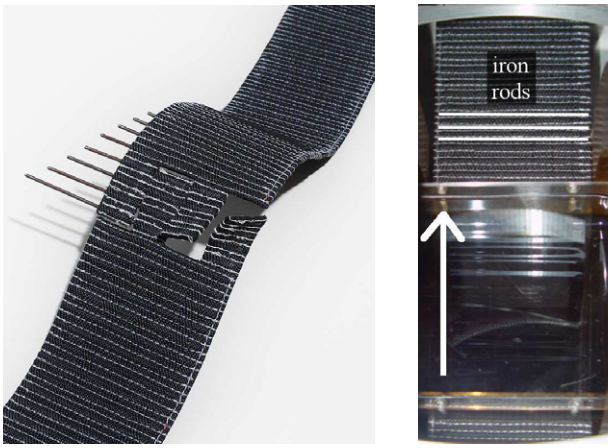


Figure 1. Sketch of the experimental setup.

inner dimensions are 100 mm (x), 120 mm (y), 25 mm (z). It is filled with ferrofluid up to $z = 3$ mm. A Helmholtz pair of coils generates a constant magnetic induction which is homogeneous with a deviation of 1% over the size of the container. In addition we apply a small (5%) spatiotemporal modulation of the magnetic field with the spatial periodicity of the critical wavelength $\lambda_c = 2\pi\sqrt{\sigma/(\rho g)}$, which propagates parallel to the surface with constant velocity v. Then traveling waves at the surface of the fluid are formed with the same wavelength and speed as the driving. The field modulation



(a) (b)

Figure 2. Experimental realization of the spatiotemporal excitation: conveyor belt with the iron rods (a), and view of the ferrofluid container from above (b). Note the surface undulations in the upper part of the container due to traveling-stripe forcing.

is realized by a 'conveyor belt', made from a textile band which harbors periodically placed iron rods. The lateral distance between neighboring rods was selected to be as close as possible to the critical wavelength $\lambda_c = 9.98$ mm. We achieved 9.3 ± 1 mm. The rods have a length of 70 mm and are made of welding wire with a diameter of 1.0 mm. The vertical distance between the symmetry axis of the rods and the surface of the fluid at rest is 8 ± 0.5 mm. The band is driven by an electric motor which allows us to vary the velocity up to 30 cm s^{-1}. Figure 2(a) shows the bare belt and (b) its location and its effect within the setup. The iron rods amplify the magnetic field locally due to their higher susceptibility; thus the magnetic field strength varies along the driving direction. As demonstrated in figure 3, the excitation profile is approximately sinusoidal. We investigate the response of the ferrofluids APG 512a and EMG 909 (Ferrotec Co.). The parameters of those fluids are listed in table 1.

For measuring the amplitudes we direct a beam of a helium–neon laser onto the surface of the magnetic fluid in the middle of the container, from where it is reflected to a screen. The beam position on the screen is acquired via a charge-coupled-device (CCD) camera and reveals the slope of the surface at the incident point of the laser beam. We extract the height of the undulations A by assuming that the surface modulation is sinusoidal, which is valid in this case of only small deformations. When the conveyor belt moves under the vessel, the fluid ridges travel with the iron rods and the position of the reflected beam oscillates.

Like in figure 3, the surface undulations are approximately sinusoidal. Deviations stem from inaccuracies in the spacing of the iron rod lattice, and fluctuations in the driving velocity of about 0.1%, which results in a phase jitter of $\pm\pi/2$. The noise spectrum extends above and below the modulation frequency, and can be removed by applying a bandpass filter centered around the mean frequency of the moving iron rods.

The amplitude of the surface waves is determined from the Fourier spectrum from the intensities of the mode

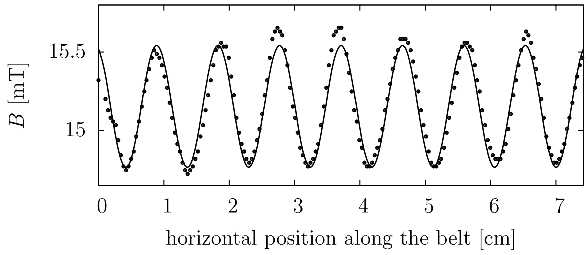


Figure 3. Modulation of the vertical magnetic field along the conveyor belt (circles), and its sinusoidal fit (solid line) by $B(x) = B_0 + \Delta B \sin(k_c x)$, with $B_0 = 15.15$ mT and $\Delta B = (0.39 \pm 0.01)$ mT.

Table 1. Parameters for the ferrofluids APG 512a Lot 083094CX and EMG 909 Lot F050903B. B_c is computed from the material parameters, while $B_{c,m}$ corresponds to the results obtained in this paper.

Property		APG 512a	EMG 909
Density	ρ	1.26 g cm^{-3}	1.0047 g cm^{-3}
Surface tension	σ	30.57 mN m^{-1}	24.51 mN m^{-1}
Initial susceptibility	χ_0	1.172	0.760
Viscosity	η	120 mPa s	4.7 mPa s
Critical wavelength	λ_c	9.98 mm	9.81 mm
Computed critical induction	B_c	17.22 mT	25.95 mT
Measured critical induction	$B_{c,m}$	15.88 mT	23.64 mT

corresponding to the mean passage time of one rod and the two neighboring modes. The spectrum is calculated within a passage time of 314 rods, i.e. one complete cycle of the conveyor belt. Because the data can be continued periodically, a window function for the elimination of artifacts is not needed.

3. Experimental results

We have measured the amplitude of the fluid waves below the critical field of the Rosensweig instability as a function of the driving velocity and the applied magnetic field for two different magnetic fluids. For each fluid we selected about ten different magnetic inductions in a range where visible undulations occur. The results for three representative values of B are displayed in figure 4. For slow driving velocities the amplitude of the undulations resembles that of the static case. For increasing velocity, the amplitude of the traveling waves passes through a maximum and decays for high driving velocities. When increasing the magnetic field the undulations become remarkably higher and the maximum shifts to lower driving speeds. When the critical field is reached, the undulations are replaced by Rosensweig spikes. Note the different responses of the highly viscous (a) and the less viscous fluid (b).

Below the Rosensweig threshold B_c, the amplitude response to this spatiotemporal driving can be modeled as a damped forced harmonic oscillator

$$A(v) = \frac{A_0 v_p^2}{\sqrt{(v_p^2 - v^2)^2 + 4\gamma^2 v^2}}. \tag{1}$$

Here $A(v)$ denotes the amplitude dependent on the driving velocity v, where A_0 is the amplitude at zero velocity, v_p

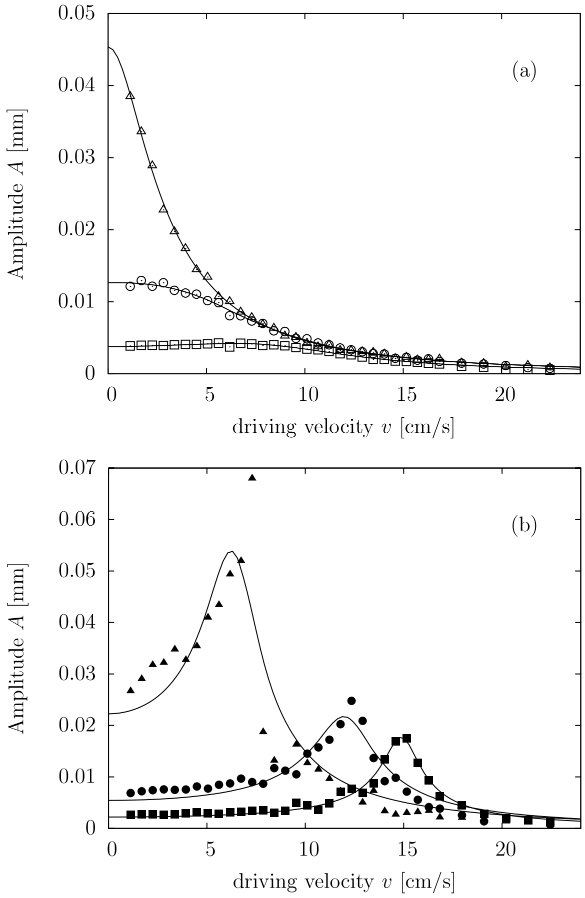


Figure 4. Amplitude of the waves versus the driving velocity for varying magnetic induction for the ferrofluids APG 512a (a) and EMG 909 (b). The triangles mark the highest induction, the circles an interim value and the boxes the lowest field. The data have been captured at the following magnetic inductions: △: 15.0 mT, ⊙: 13.4 mT, ⊡: 10.4 mT, ▲: 20.9 mT, ●: 14.9 mT, ■: 10.4 mT. The solid lines display a fit with (1).

denotes the phase velocity of the unforced surface waves and γ the damping constant. As displayed in figure 4, the experimental data are well captured by fits to (1).

The viscosity determines the damping constant in our model (1). This is corroborated by figure 4(a), where the curves of the highly viscous APG 512a show an overdamped behavior. In contrast, the fluid EMG 909, with a 25 times smaller viscosity, displays a clearly visible maximum for all magnetic inductions (see (b)).

The resonant propagation speed v_p can be calculated from the dispersion relation. For infinite layer thickness and inviscid fluids the surface waves on ferrofluid are described by the plane dispersion relation

$$\omega^2 = gk + \frac{\sigma}{\rho}k^3 - \frac{1}{\rho}\frac{(\mu_r - 1)^2}{\mu_0\mu_r(\mu_r + 1)}B^2k^2 \qquad (2)$$

put forward by Cowley and Rosensweig (1967). In our experiment we intentionally constrain the wavenumber of the surface waves by means of the iron rods to k_c. From (2), we get a phase velocity $v_p = \frac{\omega}{k_c}$ which depends on the magnetic

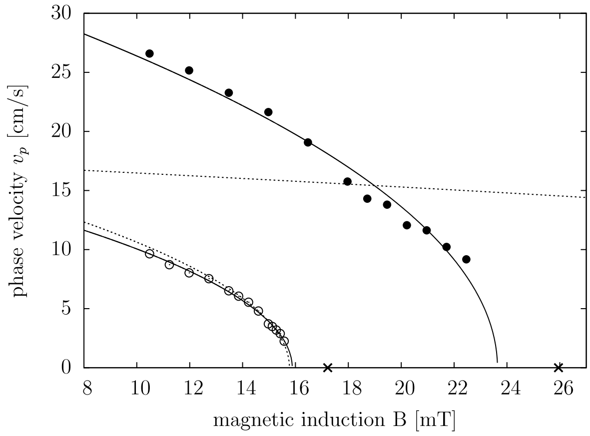


Figure 5. The resonant velocity for APG 512a (empty circles) and EMG 909 (full circles) extracted from the fit using (1) plotted against the magnetic induction. The curves display fits by (3). The dashed curve results from a one-parameter fit taking into account the material parameters whereas the solid lines display a two-parameter fit. The crosses give the computed values of B_c according to table 1.

induction:

$$v_p = \sqrt{\frac{2}{\rho}\sqrt{g\rho\sigma} - \frac{1}{\rho}\frac{(\mu_r - 1)^2}{\mu_0\mu_r(\mu_r + 1)}B^2}$$
$$= \sqrt{\alpha\frac{B_c^2 - B^2}{B_c^2}}, \qquad \text{with } \alpha = 2\sqrt{\frac{g\sigma}{\rho}}. \qquad (3)$$

If the driving velocity coincides with this velocity, the system is in resonance and the amplitude exhibits its maximum. The phase velocity v_p for different magnetic fields is experimentally obtained by fitting (1) to the data. The results are shown in figure 5. With increasing B the velocity v_p decreases and eventually becomes zero at the measured critical induction $B_{c,m}$. At this induction the surface exhibits non-propagating undulations, which is a manifestation of the Rosensweig instability.

Fitting the data with (3), where α is computed from the material parameters from table 1 and only B_c is adjusted, yields the curves marked by dashed lines. Fitting both B_c and α gives the solid lines. The values corresponding to the latter procedure are included as $B_{c,m}$ in table 1.

4. Discussion and conclusion

Applying a novel type of magnetic traveling-stripe forcing with $k = k_c$ to the subcritical regime of the Rosensweig instability we measured the response of surface waves at different driving velocities v. For a driving at the phase velocity of free surface waves a resonance phenomenon is observed, which can quantitatively be described as that of a damped harmonic oscillator. The resonant velocity v_p depends on the applied magnetic induction and decreases to zero when the critical induction B_c is approached. The functional dependence of v_p is essentially captured by the dispersion relation for an inviscid magnetic layer of infinite depth (Cowley and Rosensweig 1967).

When comparing the values B_c as computed from the material parameters and the fitted values $B_{c,m}$, the latter are shifted by 8% and 9% to lower inductions. This deviation is larger than the discrepancy of 3% obtained from amplitude measurements in the supercritical regime of the Rosensweig instability (Gollwitzer *et al* 2007). This may partly be explained by the limited resolution in the immediate vicinity of B_c. More importantly the true value for B_c can only be obtained in the limiting case for vanishing modulation amplitudes ΔB, while ΔB in our case is approximately as large as the deviation $B_c - B_{c,m}$. In addition, ansatz (3) takes into account neither the finite viscosity and layer depth (see Lange *et al* 2000) nor a nonlinear magnetization law as utilized by Gollwitzer *et al* (2007) and Knieling *et al* (2007). Further deviations may stem from the difficulty in producing an ideal rod lattice, leading to a scatter in the wavelength λ by 4.3% (rms). From figure 4 we conclude that the thin ferrofluid EMG 909 is more susceptible to these irregularities than the highly viscous fluid APG 512a. This may also be the reason for the difficulties with the one-parameter fit for this type of ferrofluid.

There are several possible applications of our spatiotemporal forcing. On the one hand, it can be used to create a volume flow (Kikura *et al* 1990). A theoretical estimate for this flow was recently provided by Zimmermann *et al* (2004) and is awaiting an experimental proof. This method of pumping is an alternative to that proposed by Mao and Koser (2005) and to that proposed by Liu (1998); the latter was realized by Krauss *et al* (2005).

Further, the spatiotemporal forcing opens up new possibilities for the general study of the Rosensweig instability. As an advantage, it may be used to fix the wavelength to preselected values. This represents an important difference from a previous study by Reimann *et al* (2003), where the critical scaling of a freely propagating wave has been measured, and a dependence $k(B)$ had to be taken into account. Utilizing a k in the proper range may give an elegant access to the anomalous branch of the dispersion relation, which has only recently been studied by gravitational excitation in combination with different filling levels (Embs *et al* 2007). Moreover our magnetic forcing may be applied to the supercritical regime of the Rosensweig instability, where new resonances between hexagonal, square, or stripe-like patterns and the traveling-stripe forcing are predicted (Rüdiger *et al* 2007).

Acknowledgments

We thank Christopher Groh and Klaus Oetter for technical assistance with the setup. Financial support by *Deutsche Forschungsgemeinschaft* under grant Ri 1054/1-4 is gratefully acknowledged.

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